

Quantization of the space–time based on a formless finite fundamental element

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Abstract

The concept of the space (space–time) of the formless finite fundamental elements (FFFE) is suggested. This space can be defined as a set of coverings of the continual space by non–overlapping simply connected regions of any form and arbitrary sizes with some probability measure. The average sizes of each fundamental element are equal to the fundamental length. This definition enables to describe correctly the passage from the space of the formless finite fundamental elements to the continual space in the limit of zero value of the fundamental length. FFFE space–time functional integral construction is suggested. A wave function of a separate FFFE and the overall wave function of a manifold are introduced. It is shown that many other constructions of the discrete space–time (the Regge coverings, the lattice space–time etc.) are the special cases of this space–time.

A vacuum action problem is analyzed. One term of this action is proportional to the volume of a fundamental element. It is possible to direct the way for this term to yield the Nambu–Goto action in consideration the string as one–dimensional excitation of a number of FFFE. Fermionic and bosonic fields in the space–time of FFFE are excited states of elements. Space–time supersymmetry leads to supposition that the maximal possible number of fermionic excitations at one FFFE is equal to the number of elements in all space–time. The compactification in this space–time means the condition of the neighbour elements absence in compactified dimensions.

I. Introduction.

In the classical theory the space–time is a continuum, where the fundamental elements are points. In continual geometry all the geometrical objects are sets of points. Scalar, vector and tensor quantities are the functions of point coordinates. Mathematical analysis operations (limits, derivations etc.) are defined at a point.

In the Dirac quantum mechanics and the quantum field theory the space–time is also represented as a continuum. In the classical theory of gravitation the space–time is the continuum. But the quantum analysis of the space–time properties provides some arguments counting in favour of the existence of the minimal length, that can be measured by physical methods. The Heizenberg uncertainty relations application to the process of small distances measuring yields the inequality [1]

$$(\Delta L)^2 \geq 2 l_{pl}^2 \quad (1)$$

where $l_{pl} = (G^{-1} \hbar^{-1} c^3)^{\frac{1}{2}} \simeq 10^{-33} sm$.

The analysis of the space–time properties on the Plank distances leads to the idea of "space–time foam" [2-3]. The postulate about a fundamental length (l_f) existence is realized in the concept of the quantized (discrete) space–time, that consists of the fundamental elements with the finite sizes [4-16]. The lattice space–time with the fixed lattice is most commonly investigated [9-13].

But the space–time with the fixed lattice consideration leads to several problems. The first and seemingly the most essential problem is the passage from the lattice space–time to the continuum in the limit $l_f \rightarrow 0$. The lattice space–time has the power of a countable set. Any subdivision of a lattice yields a set with the same power. Thus in the limit $l_f \rightarrow 0$ any lattice space–time with any subdivision remains a set with the power of a countable set. The second, the resulting equations in the lattice space and other spaces with determined fundamental element form depend on the form of a fundamental element. The third, the equations in the lattice space are non-invariant under

the continual symmetry operations.

Regarding the quantum ideology all physical quantities don't have any determined values. Thus if the quantum concept is applied to the space-time consistently, then it is possible to operate on only the average sizes of the elements. In this sense all the determined forms of the space-time fundamental element including stereohedra are not consistently quantum description of the space-time. Consistently quantized space-time can be formed by the fundamental elements without any determined forms and sizes only. Nothing but average sizes of each fundamental element in consistently quantized space-time can be determined.

Discrete geometry has been developed in the direction of the formless fundamental element in the last thirty years. The Regge calculus [16] and the space-time foam idea [2-3] made the first steps on this way. In refs. [14] the stereohedra space is investigated. This space fundamental element has some set of forms. Random lattice field theory is analyzed in [15]. In ref. [7] develops the topological approach to quantized space-time calculations. Quantum configuration space investigated in [8] is the method of quantized space-time description based on the not-fixed "floating lattice".

In the present work the concept of the space of the formless finite fundamental elements is suggested. In this concept the problem of the passage to the continuum in the limit $l_f \rightarrow 0$ and several other problems of the lattice space-time may be solved.

Notations

n – the dimensionality of a space or a space-time

η_{ik} – the metric tensor of the plane space or space-time

g_{ik} –a metric tensor of the Riemannian space or space-time

$\{a\}$ – a set of coordinates in the space or the space-time of FFFE's

l_f – the fundamental length

l_{pl} – the Plank length

Abbreviations

FFFE – a formless finite fundamental element

FL – the fundamental length

Below $l_f \simeq l_{pl}$ is supposed.

II. Geometry of the space of the formless finite fundamental elements.

Introduce the postulates, that differ the geometry with a formless finite fundamental element from the continual geometry.

Postulate 1. The fundamental length (l_f) is a geometrical quantity of length dimension that means the quantum limit of measurements accuracy in the space (space–time).

Postulate 2. The space (space–time) consists of the fundamental elements that have finite sizes.

Postulate 3. The formless finite fundamental elements have average sizes by order fundamental length at every dimension. All physical and geometrical quantities are described as fields defined on a set of FFFE's.

Postulate 4. All physical and geometrical quantities don't depend on the form of concrete FFFE. They can depend on average geometrical characteristics of FFFE's only.

In the text below n is the dimensionality of the space (or space–time) of FFFE's. This dimensionality is equal to the dimensionality of the continual space. Introduction of the space of FFFE's axiomatically allows to construct the mathematical objects and operations in this space without consideration of the continual space objects and operations.

On the one hand these postulates are the most probable to be obtained consistently from analysis of the space–time quantum properties at the small distances. On the other hand the axiomatical definition of the quantized space–time properties could itself leads to the quantization phenomenon and

the quantum field theory as the consequences of the space–time structure.

The postulates 1 and 2 are identical to the postulates of lattice and stereohedra geometries. But the postulates 3 and 4 specifies the geometry with a formless finite fundamental element from other geometrical construction of the discrete space.

The postulate 4 allows to define the space of FFFE as the set of coverings of the continual space by any number of non-overlapping simply connected regions of any form and arbitrary sizes. This set is provided with the probability measure, i.e. each covering contributes to the space with some probability. This measure enables the calculations based on this coverings set (see the sections III, IV). The average values of sizes of FFFE are equal to l_f , and the average number of FFFE localized in the continual space region by the volume V is $N = [V(l_f^n)^{-1}]$. But the configurations with greatly different from l_f FFFE sizes also have the finite probabilities, for example, the configurations with one fundamental element that expands on all the space–time (or investigated manifold), or the configuration of continual space–time region itself, i.e. covering this region by points. This set of coverings have the power of continuum. Therefore limit passage from the space of FFFE to the continual space can be carried out correctly.

In the space of FFFE the coordinates can be introduced in the region, consisting of a number of FFFE. The space coordinates on one fundamental element don't have a determined meaning in the space of FFFE and can be of auxiliary character. The coordinates introducing on all the set of FFFE is difficult problem due to a number of fundamental elements (regions from coverings) on one manifold is variable. The coordinates can be introduced with sufficient correctness only on the set of configurations in which all sizes of all elements are about equal to l_f , all m –dimensional areas are about equal to l_f^m , and all elements volumes are about equal to l_f^n .

All geometrical operations in the space of FFFE are determined with accuracy $O(l_f^k)$. Thus generators of the rotation group in the space of FFFE are rotations on a finite angle. Evidently the infinite small transformation

like the ones in the continual space cannot be the space transformation operations defined in the space of FFFE, because the infinite small transformation doesn't cause any modifications in the set of FFFE. The translation group generators are the translations on a finite distance (by order l_f).

This accuracy limit of operations determination helps to solve the same problems arising in the lattice space-time consideration. Thus the lattice Dirac equation is relativistic invariant with averaging on the continual rotation group only [9, 10]. In the space-time of FFFE this problem is solved at the postulate level, because this averaging is the consequence of the postulates 1-4.

The many other constructions of the discrete space-time (i.e. the Regge coverings, the lattice space-time, the random lattice space-time, the stereohedra space-time) are the special cases of FFFE space-time with the special choice of the probability measure. Thus the lattice space-time is the set of coverings with the probability measure that is equal to zero for the configurations differ from the coverings by n -dimensional cubes with identical sizes. The random lattice space-time is the set of coverings with the probability measure that is not equal to zero for the coverings with the rectangular lattice of the variable step.

The mathematical operations and the physical equations in the space and the space-time of FFFE could be obtained by two methods. The first one is based on the known operations and equations of the continual space. The operations and equations in the FFFE space are the ones for average values, that are calculated by the method of functional integrals. This method is considered in the section III.

The second method is the postulative introduction of operations and equations in the space of FFFE that requires the definitions of invariant objects on the set of FFFE. These objects depend on the place of concrete FFFE among the other elements and average sizes and volume, in the same time they don't depend on the form and the sizes of concrete FFFE.

Invariant objects defined on each fundamental element must be the invari-

ants of the complete space transformation group. One can note that the plane space of FFFE has a specific transformation operation that is absent in the continual spaces and in the FFFE curved space. This operation is rearranging of elements. Regarding physics the plane space–time can be free of particles only, when with geometrical consideration the properties of the plane space are identical in all the space. Therefore any number of elements are able to change their localization in any order, and space of FFFE or the manifold of this space is transformed into itself. In the Riemannian space this operation isn't symmetry operation due to the coordinate dependence of the connection and different values of excitations probabilities on different fundamental elements. Due to this rearranging symmetry the plane space and the Minkowski space–time of FFFE are completely stochastized because any FFFE localization region isn't exactly determined. Riemannian space and space–time with particle–like excitations are not stochastized since geometrical and physical properties is chosen from one element to other.

III. Functional integral in the space–time of FFFE.

In the previous section the calculations method with use of invariant structures of the FFFE space was discussed.

The other way of obtaining the physical equations and mathematical operations in the space and the space–time of FFFE is calculations with use of a continual (functional) integral. In agreement with the central idea of the continual integral theory the calculation of quantum quantities is the integrating over all possible configurations of the space (space–time) of FFFE (i.e. coverings of the space (space–time)) with into account the corresponding probability measure taken.

Consider the general construction of a functional integral. In the plane space it is:

$$Z = \int \mathcal{D}V e^{-S(s_i)}, \quad (2)$$

where $\mathcal{D}V$ is a measure in the set of coverings, $S(s_i)$ is the plane space (space–

time) vacuum action, s_i is the set of element parameters (sizes, areas, volume). Here integrating is over all coverings of the continual space (space–time) by non-overlapping simply connected regions of any forms and any sizes (see below). Average value of a function on a separate FFFE is defined by

$$\langle f(\{a\}) \rangle = \frac{\int \mathcal{D}V e^{-S(s_i)} f_{\{a\}}(x^i)}{\int \mathcal{D}V e^{-S(s_i)}} \quad (3)$$

where $f_{\{a\}}(x^i)$ is values of the function f at regions of coverings set which forms the element $\{a\}$, $f(x^i)$ is a function defined in the continual space (space–time).

In the curved space (space–time) a vacuum functional integral is

$$Z = \int \mathcal{D}V \mathcal{D}g_{ik} e^{-S(s_i, g_{ik})} \quad (4)$$

Here S is the curved space (space–time) action. Full Riemannian space–time action includes particle terms (see in detail in the section VI). This action is the one of the space–time with excited states, i.e. vacuum action + action of excitations. Average value of an operator in the Riemannian space (space–time) is represented by

$$\langle A(\{a\}) \rangle = \frac{\int \mathcal{D}V \mathcal{D}g_{ik} \mathcal{D}\varphi_m A_{\{a\}}(g_{ik}, \varphi_m, x^i) e^{-S(g_{ik}, \varphi_m, s_i)}}{\int \mathcal{D}V \mathcal{D}g_{ik} \mathcal{D}\varphi_m e^{-S(g_{ik}, \varphi_m, s_i)}} \quad (5)$$

where φ_m is any fields defined in the continual space. Thus the functional integrating operation is the one of the averaging over all configurations that form the space of FFFE with the corresponding action. This construction is similar to the functional integral over surfaces in the Polyakov superstring theory [20].

Integrating in the functional integral is over all possible coverings of the continual space by non-overlapping simply connected regions with arbitrary sizes and forms. The application of FFFE functional integrals require the information about the action S . This problem is discussed in the section IV for the vacuum case.

In the space of FFFE the wave function of each FFFE could be introduced (see the remark about the coordinates introducing in FFFE space in

the section II). This function squared determines the probability of finding the concrete FFFE in the state with an average localization point \vec{r} , the volume V , total m -dimensional areas S_m and sizes l_i . Denote it $\psi_{\{a\}}(\vec{r}, V, S_m, l_i)$. Here $\{a\}$ is the set of fundamental element coordinates in the space of FFFEs. This wave function squared $|\psi|^2$ is a density of probability in the set of coverings, i.e. $|\psi|^2 d\sigma$ is the probability of finding the element with FFFE space coordinates $\{a\}$ in the state with continual parameters l_i, S_m, V . Here $d\sigma$ is a measure in the set of coverings. In principle any physical quantities could be found as matrix elements

$$\langle A(\{a\}) \rangle = \langle \psi_{\{a\}}(\vec{r}, V, S_m, l_i) | \widehat{A} | \psi_{\{a\}}(\vec{r}, V, S_m, l_i) \rangle \quad (6)$$

where A is a function in the continual space. The notation $\langle A(\{a\}) \rangle$ means the summing over all regions of coverings set which form the element with the coordinates set $\{a\}$ in the space (or space-time) of FFFEs. Summarized quantities are average values of $\langle A \rangle$ on the each configuration element, multiplied on $|\psi|^2$.

The state of the space of FFFEs is a covering of the continual space. It is also possible to introduce the wave function of a space manifold or all space wave function (see the discussion of the corrections below):

$$\Psi = \prod_{\{a\}} \psi_{\{a\}}(\vec{r}, V, S_m, l_i) + com(\{\psi_{\{a\}}\}) \quad (7)$$

This Ψ describes a covering of the continual space by non-overlapping simply connected regions. $|\Psi|^2$ is the probability density in the set of coverings. This function describes the state of the space-time without particles, i.e. particle-like excited states of FFFE. However, the vacuum itself has excited states, where the elements sizes and localization points differ greatly from the average values. Functions $\psi_{\{a\}}$ are not independent because they describe non-overlapping regions. Therefore the expression (7) contains the second term that is determined by the commutation relations.

IV. The Minkowski space–time action.

The functional integral construction considered in the previous section requires the information about the action. In this section the case of the pure vacuum is discussed. Regarding physics the case of the vacuum is described by the plane Minkowski space–time. Any nonvacuum excitation, including the virtual particle vacuum polarization and the all space constant fields leads to arising of the connection and the curved structure of the space–time.

Suppose that one term of the space–time action is proportional to the volume of FFFE:

$$S_V = \int_{FFFE} \alpha \sqrt{-g} dV \quad (8)$$

Here dV and $\sqrt{-g}$ are continual space values. Integrating in (8) is over one region from some covering of the continual space–time. This term is analogous of "space–time foam" action proportional to the volume [2]. But this term of an action is insufficient for describing the equilibrium configuration of the space–time of FFFE. Total actions (8) of all configurations from the set of coverings are equal.

Consider the problem of the space–time action minimum. On the one hand as a rule in the method of functional integrating the minimum of considered action describes the corresponding classical system (a moving pointlike particle in the Feynman integral, the space–time with the classical value of a metric tensor in the integrating over space–time metrics in quantum gravity etc.). But in the case of the space–time the classical system is the continual space–time. It means that the space–time action must have the minimum in the configuration with infinite number of fundamental elements which are points. However, the action minimum in the continual space–time configuration, and as the consequence the finite value of the probability measure of this configuration, leads to divergence of some integrals, for example, the average value of a number of fundamental elements on a continual manifold. It is seems that the probability measure must be small in the configurations with small number of fundamental elements.

On the other hand the equilibrium state of the FFFE space-time is the one with average FFFE sizes at a fundamental length. The more correct approach to the equilibrium action problem is to find the fundamental length, firstly, as the average value of a FFFE size, secondly, as realizing of some quantized action minimum. It means that the average number of FFFE in a manifold is determined by the solution of the action minimum problem.

The complete expression for the space-time action, meeting this requirement, must contain other terms besides the volume term. The possible term is the one proportional to the total $n - 1$ - dimensional area of FFFE. In this supposition the vacuum space-time action is

$$S = \alpha \sum_i V_i + \beta \sum_i (S_{n-1})_i \quad (9)$$

where \sum_i is summarizing over all FFFE.

The constants in the action expression can be product of the universal constants only. Thus in the four-dimensional space-time

$$\alpha = A \hbar^{-1} G^{-2} c^6 \quad (10)$$

where A is a numerical factor.

Let us direct the way, on which the Nambu-Goto term of the string action [21] might be obtained from the space-time action (8). The expression of the action of a space-time element for this analysis is required. This action is the average value of an action S with the functional integrating (3) using. This action is denoted by S_{FFFE} :

$$S_{FFFE} = \langle S_{\{a\}} \rangle \quad (11)$$

or

$$S_{FFFE} = \int \sqrt{-\eta} dx^1 dx^2 dx^3 dx^4 \quad (12)$$

for the four-dimensional space-time. This construction also could be introduced axiomatically as the invariant structure (see the section II).

A string in the space-time of FFFE can be considered as an excitation of a number of FFFE forming one-dimensional space-like curve (in the meaning

of FFFE space–time). In the own reference frame the action of this excitation is represented in the form

$$S = A \hbar^{-1} G^{-2} c^6 \int_{FFFEs} \sqrt{-\eta} dx^1 dx^2 dl d\tau \quad (13)$$

where τ is the own time, l is the own space–like coordinate of an excitation, x^1, x^2 are the transverse space–like coordinates. Here integrating is over a set of FFFE, participated in the excitation propagation. After integration over transverse coordinates we might obtain (γ is the two–dimensional metric tensor determinant):

$$S = A G^{-1} c^3 \int dl d\tau \sqrt{-\gamma} \quad (14)$$

i.e. the Nambu–Goto action for a string. In (14) the equality of the FFFE average size on the one dimension to the fundamental length is taken into account. This result is not completely correct due to the transformation problem of the four–dimensional metric tensor determinant η in (13) to the two–dimensional one γ in (14) and absence of the correct definition of one–dimensional integrating. In this concept the p –branes are considered as the p –dimensional space–like excitations of FFFE, and the volume term (13) of FFFE space–time excitation yields the bosonic term of p –branes action analogically.

V. The compactification in the space–time of FFFE.

In the concept of the FFFE space–time the multidimensional space–time with the motion possible on four dimensions only can be described without any special compactification procedure. The multidimensional space–time with average sizes on the higher dimensions by order l_f can be constructed using the postulates about the absence of neighbour elements for all ones on all the dimensions without four. These average sizes on the higher dimensions are l_f despite the configuration with sizes, which are significantly more l_f on these dimensions contributes to the space–time structure.

But this compactification description is not satisfying as it requires the introducing the special postulate. The deeper approach to the compactification problem is to formulate the neighbour element absence requirement, caused space–time action structure analysis or some geometrical requirement. Erenfest’s investigations about stability of systems with Coulomb interaction shows that 4–dimensionality of the real space–time connect with particle action, more precisely, with the interaction part of particles action. This way the case $n = 4$ of the FFFE space–time dimensions, on which the motion is possible, yields the minimum of an action of n –dimensional curved space–time.

VI. Physical fields in the space–time of FFFE.

Fermionic and bosonic fields are excited states of FFFE. As any quantum particle, excluding a free particle, has a wave function with different values $|\psi|^2$ in different points of the space–time, the space–time with excitations couldn’t be the Minkowski space–time. Different values of $|\psi|^2$ in different FFFE violate the Minkowski space–time specific symmetry under rearranging of any number of FFFE (see section III). Therefore two interacting particles in the space–time result in the curved space–time with changeable curvature. One particle in all space–time or the uniform vacuum polarization leads to the particle–like excited curved space–time with the constant curvature.

Particle–like excitations of the FFFE space–time are finite in each FFFE of considered manifold. The state of FFFE with particles excitations in the approximation of ininteracting particles is described by a wave function

$$\psi_{\{a\}}^{ex} = |\{a\}, \psi_i >, \quad (15)$$

where $\{a\}$ is a set of coordinates in the FFFE space–time, ψ_i is one-particle wave functions values in this FFFE.

A wave function $|\{a\}, 0 >$ is the sum of all excited vacuum states. In the classical space–time $|\psi|^2 dV$ is interpreted as a probability of a particle localization in the volume dV . In the FFFE space–time the interpretation

$|\psi_{\{n\}}|^2 dV_{fund}$ is a probability of finding this FFFE in the excited state with the set of quantum numbers (charges) $\{n\}$. This probabilities equality means the influence of the other FFFE's with excited states on the excited state of this FFFE. In other words, the space-time of FFFE's is the self-organizing system.

The states of a manifold of the space-time of FFFE's is described by an overall wave function, that could be obtained by summarizing of each FFFE wave function with the commutation relations taken into account. Functional integral construction in the curved space-time requires to include particles terms of the space-time action in consideration.

It is to suppose that the number of possible fermionic excitation in one FFFE is finite. In this case the space-time supersymmetry leads to the supposition about the equality of a number of possible fermionic excitations in one FFFE and a number of FFFE's in all the space-time.

VII. Final remarks.

Ideologically the suggested concept is consistent geometrical approach to the physics of fundamental particles and interactions. This concept may help to solve some problems of lattice space-time geometry due to more consistent quantum approach to the space-time structure problem. In this space-time the particles are geometrical objects - excited states of space-time elements. The superstrings can be constructed as propagating excitations of space-time elements. At this approach the Nambu-Goto action term is considered as a result of this excitation volume space-time action term analysis. Consideration of the superstrings as the excitations of the quantized space-time is the step to the understanding of the superstrings properties at the Plank distances. With this superstrings and p -branes consideration all these objects are identical at the Plank distances because the excitation of one element doesn't have dimension in the sense of FFFE space-time.

But some problems of elementary particle and field theories are not obvious

in this concept, i.e. the appearance and the role of gauge invariance in the FFFE space-time, appearance of charges of Riemannian space-time excited states, posing of the cosmological evolution problems and some others.

In conclusion it is some words about axiomatical introducing of the quantized space-time (see the section II). Certainly the author can't be sure that the postulates 1 - 4 are most correct, complete and minimal system of the quantized space-time axioms. It is not improbable that the postulate 1 about the fundamental length existence is the consequence of some other axioms system, and at the same time the quantized space-time properties are defined by the axioms introducing the set of FFFE's operationally. But it is to be noted that on this way the uncertainty relations, quantum field theory and quantization phenomenon itself are the consequences of these quantized space-time axioms system. In particular, it is supposed that the Dirac term of lagrangian density can come as a consequence of coordinates matrix introducing [18] and the Weyl structure of FFFE curved space-time. The Weyl structure of FFFE curved space-time is connected with the finite accuracy of any geometrical operations. Thus the vector length at its parallel transport from one FFFE to other is determined with an accuracy of l_f . Therefore the FFFE curved space-time is the Weyl space-time automatically, and the Weyl distortion of the Riemannian structure of this space-time is caused by quantized structure of the space-time principally.

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